

Minimal quartet-metric gravity: beyond Λ CDM per scalar graviton as the unified dark matter and dark energy

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Abstract

In the framework of the minimal quartet-metric gravity/*systogravity*, a scalar graviton/*systolon* is stated as a universal dark component, with supplementary manifestations in the different contexts either as dark matter or dark energy. An ensuing extension to the standard Λ CDM model is developed. A modification of the late expansion of the Universe, with an attractor of a scalar master equation defining an effective cosmological constant, which supersedes the true one, is proposed. A new partial solution to the cosmological constant problem is discussed.

1 Introduction

The present-day standard scenario for evolution of the Universe is given by the Λ CDM model or, otherwise, the standard cosmological model (SCM). The scenario includes a primeval inflation followed by the thermodynamic expansion with the subsequent late acceleration due to a cosmological constant (CC). Despite the impressive successes of SCM, the ever-growing evidences from the astrophysical observations may indicate a vital necessity of going beyond such the model in the future.¹ SCM is based on General Relativity (GR) as a working tool. Thus, going beyond SCM may, in particular, imply going beyond GR.² In this vein, in Ref. [4] there was developed an extension to GR, the so-called *quartet-metric* GR (QMGR), describing in a general case the scalar-vector-tensor gravity in an explicitly generally covariant (GC) fashion. This theory was proposed for unifying the tensor gravity with a gravitational dark content (DC), the latter manifesting itself as the dark matter (DM) and dark energy (DE). In the full theory, DC consists of the three dark components: the (massive) scalar, vector and tensor gravitons. It is reasonable however to start investigating DC restricting himself just by the massive scalar graviton in addition to the conventional (massless) tensor one. In such the minimal version, the theory was previously applied in the context of DM for some cosmic dark structures (DSs) [5]. In the present paper, the minimal version of the theory is further elaborated in the context of DE for the Universe as a whole.

In Sect. 2, the minimal QMGR, treated as a part of the full theory, is concisely presented, with the aim of applying it in cosmology as a basic gravity theory superseding GR. The appearance in the effective scalar-graviton potential, in excess of the Lagrangian potential, of a spontaneous contribution, which proves to be crucial, is explicitly demonstrated. In Sect. 3, the scalar graviton is shortly recapitulated as DM and worked-out in more detail as DE. An extension to the standard Λ CDM model due to the omnipresent in the Universe

¹For the primeval inflation, see, e.g., [1]. For the standard Λ CDM model and beyond, see, e.g., [2].

²For the gravity beyond GR, see, e.g., [3].

scalar-graviton field is developed. In particular, a modification of the late expansion of the Universe, governed by an autonomous scalar equation, is proposed. It is argued that the general solution to the equation asymptotically approaches an attractor defining, in turn, an effective CC superseding the true one. At last, an ensuing hereof partial solution to the CC problem is discussed.

2 Systogravity and scalar graviton

2.1 Systogravity Lagrangian

We start with a concise exposition of the theory of the massive scalar and massless tensor gravitons to be applied in what follows to the Universe instead of GR. As a basic extension to GR, we adopt the effective field theory (EFT) of gravity, QMGR [4], given generically by a GC action:

$$S = \int L_G(g_{\mu\nu}, X^a, \eta_{ab}) \sqrt{-g} d^4x. \quad (1)$$

Here, an extended gravity Lagrangian L_G depends on the dynamical metric field $g_{\mu\nu}$ ($g \equiv \det(g_{\mu\nu})$) and a scalar-quartet field X^a , where $a, b, \dots = 0, \dots, 3$ are the indices of the global Lorentz symmetry $SO(1, 3)$, with the invariant Minkowski symbol η_{ab} . The latter, in particular, predetermines for consistency the signature of $g_{\mu\nu}$. The GC scalar fields X^a are defined up to the global Lorentz transformations and shifts $X^a \rightarrow X^a + C^a$, with the arbitrary constants C^a . The (piece-wise) invertible transformation in space-time $\hat{x}^\alpha = \delta_a^\alpha X^a(x)$ ($x^\mu = x^\mu(\hat{x})$) presents the so-called (“hidden”) *affine* coordinates.³ The latter ones are distinguished by the fact that in these coordinates an auxiliary affine connection $\gamma_{\mu\nu}^\lambda(x)$, entering L_G to ensure GC, gets “hidden”, $\gamma_{\alpha\beta}^\gamma(\hat{x}) = 0$ (though, generally, the Christoffel connection $\Gamma_{\alpha\beta}^\gamma(\hat{x}) \neq 0$).⁴ The affine coordinates are proposed to be treated physically as comoving with the vacuum.

The most general QMGR Lagrangian of the second order in the derivatives of metric is presented in [4]. Generically, such a Lagrangian describes the scalar, vector and tensor gravitons. Imposing some “natural” (in a technical sense) restrictions on the Lagrangian parameters, one can exclude the vector graviton as the most “suspicious” theoretically and phenomenologically, leaving in the leading approximation just the scalar graviton as the most “auspicious”, in the line with the conventional tensor gravity. In what follows, we restrict ourselves by such a minimal version of the theory.⁵ Containing in addition to the tensor graviton only the scalar one or, otherwise, the *systolon* [5], such a minimal QMGR from its particle-content point of view may for short be referred to as the *systogravity*.

The full QMGR Lagrangian L_G reduces for systogravity to

$$L_{sg} \equiv L_g + L_s = -\frac{1}{2}\kappa_g^2 R + \frac{1}{2}\kappa_s^2 g^{\kappa\lambda} \partial_\kappa \sigma \partial_\lambda \sigma - V_s(\sigma). \quad (2)$$

Here R is the Ricci scalar, and σ a (dimensionless) scalar-graviton/systolon field, with $V(\sigma)$ its potential. In the above, κ_g is the tensor-gravity energy scale given by the reduced Planck mass, $\kappa_g = 1/\sqrt{8\pi G_N}$, with κ_s being a similar scale appropriate to the scalar gravity. The

³The special notation $\alpha, \beta, \dots = 0, \dots, 3$ for these coordinates is used just to explicitly distinguish them among all other ones.

⁴One should not mix the (piece-wise) affine coordinates in a region with the locally inertial/Lorentzian coordinates in an infinitesimal vicinity of a point.

⁵Nevertheless, the rest of the gravity DC may prove to be needed in the future [4].

respective physical (dimensionfull) field is $\varsigma \equiv \kappa_s \sigma$. For the dominance of the tensor gravity it is assumed moreover that $\kappa_s/\kappa_g \ll 1$. Considering such an EFT at energies less than κ_s , we retain only the leading term in the derivatives of σ . On the other hand, the scalar potential V_s is a priori an arbitrary function of σ . Nevertheless, the σ -dependent part of V_s will be assumed to be suppressed due to the approximate shift symmetry $\sigma \rightarrow \sigma + \sigma_0$, with any constant σ_0 .⁶ More particularly, one has

$$\sigma = \ln \sqrt{-g}/\sqrt{-\gamma}, \quad (3)$$

where $\sqrt{-\gamma}$ is an auxiliary affine measure in addition to the metric one, $\sqrt{-g}$, as follows:

$$\sqrt{-\gamma} \equiv X = \det(\partial_\lambda X^a) > 0, \quad (4)$$

clearly independent of metric.⁷ With X^a being the GC scalars, $\sqrt{-\gamma}$ has the same weight as $\sqrt{-g}$ under the GC transformations. This ensures σ to be the true scalar field.⁸ Eqs. (3) and (4) are the key points of the dynamical theory of scalar graviton/systolon. Had σ been a conventional elementary field, this would result in a quite different theory. On the other hand, had $\sqrt{-\gamma}$ been a non-dynamical quantity, the theory would be just the semi-dynamical one [5], preserving nevertheless many features of the fully dynamical theory.⁹ For completeness, the pure systogravity Lagrangian L_{sg} is to be supplemented by a matter one, L_m , with some matter fields Φ^I .¹⁰

2.2 Systogravity field equations

The Lagrangian $L_{sg}(g_{\mu\nu}, X)$ is a marginal case of a full Lagrangian $L_G(g_{\mu\nu}, X^a)$. For this reason, though dependent explicitly only on X , the Lagrangian L_{sg} is understood as a function of the whole X^a , like the full L_G .¹¹ The resulting ambiguity in X^a (at the same X) is inessential for systogravity and may be reduced afterwards in the full theory. By this token, varying the action through $g_{\mu\nu}$, X^a and Φ^I one gets the system of the coupled field equations (FEs) for the metric, quartet and matter in the conventional notation, respectively, as follows:

$$\begin{aligned} G_{\mu\nu} &\equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{\kappa_g^2}(T_{s\mu\nu} + T_{m\mu\nu}), \\ \frac{\delta}{\delta X^a}(L_s + L_m) &\equiv -\nabla_\lambda \left(\left(\frac{\delta L_s}{\delta \sigma} + \frac{\delta L_m}{\delta \sigma} \right) X_a^\lambda \right) = 0, \\ \frac{\delta L_m}{\delta \Phi^I} &\equiv \frac{\partial L_m}{\partial \Phi^I} - \nabla_\kappa \frac{\partial L_m}{\partial \nabla_\kappa \Phi^I} = 0, \end{aligned} \quad (5)$$

⁶At that, the smallness of the constant part of the Lagrangian potential, $V_s|_{\min}$, in particular putting $V_s|_{\min} = 0$, remains to be justified as an additional condition.

⁷In the minimal QMGR, the dependence on η_{ab} disappears except for defining the signature of $g_{\mu\nu}$.

⁸In the affine coordinates \hat{x}^α , one clearly gets $\sqrt{-\gamma(\hat{x})} = 1$. Under $\sigma \neq 0$, each of the measures, $\sqrt{-\gamma}$ and $\sqrt{-g}$, can always be brought to unity separately, but not simultaneously, with a mismatch defining ultimately the scalar σ .

⁹Due to dependence of γ on the derivatives of X^a , choosing σ in (2) as an independent Lagrangian variable, as it might be tempting superficially, is an illegitimate (not a point-wise) operation resulting in the quite different classical equations. For this reason, σ can not, generally, be reduced to an ordinary scalar field.

¹⁰In the hot Universe, the theory may, generally, describe both the coherent scalar-graviton/systolon field and the respective (incoherent) thermodynamic fraction with a temperature T_s . We omit the latter fraction (if any) by putting $T_s = 0$.

¹¹Beyond the context of L_G , the dependence of L_{sg} on X^a would be just a recipe for choosing the proper field variables.

where X_a^λ is a tetrad inverse to that $X_\lambda^a \equiv \partial_\lambda X^a$. The first and the last FEs in (5) are clearly the counterparts of the gravity and matter FEs in GR, while the second FE may equally well be attributed either to the scalar matter or to gravity.¹² In the spirit of DC, the Lagrangian L_m for the ordinary matter may moreover be assumed to be explicitly independent of σ . In FEs (5), $T_{s\mu\nu}$ and $T_{m\mu\nu}$ are the canonical energy-momentum tensors, respectively, for the scalar graviton/systolon and matter obtained by means of varying the Lagrangian L_f of a fraction $f = (s, m)$ through $g_{\mu\nu}$ as follows:

$$T_{f\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_f)}{\delta g^{\mu\nu}}. \quad (6)$$

By this token, one gets

$$\begin{aligned} T_{s\mu\nu} &= \kappa_s^2 \nabla_\mu \sigma \nabla_\nu \sigma - \left(\frac{1}{2} \kappa_s^2 \nabla^\lambda \sigma \nabla_\lambda \sigma - U_s \right) g_{\mu\nu}, \\ T_{m\mu\nu} &= 2 \frac{\partial L_m}{\partial g^{\mu\nu}} - \left(L_m + \frac{\delta L_m}{\delta \sigma} \right) g_{\mu\nu}, \end{aligned} \quad (7)$$

where $U_s \equiv V_s + W_s$, with

$$W_s \equiv -\delta L_s / \delta \sigma = \kappa_s^2 \nabla^\lambda \nabla_\lambda \sigma + \partial V_s / \partial \sigma \quad (8)$$

being nothing but the scalar wave operator. Formally, the term W_s appears in $T_{s\mu\nu}$ in addition to the conventional V_s because of σ being not independent of $g_{\mu\nu}$. Physically, this off-shell contribution may be attributed to the “inertia” of the vacuum revealing ultimately through the dependence on X^a .¹³ While due to GC all the coordinates, including the affine ones, are geometrically equivalent, the physical interpretation in them may look quite differently (for the expanding Universe, see, later on). At last, precisely the off-shell term W_s crucially distinguishes the scalar graviton/systolon from its ordinary scalar counterpart, providing an ultimate reason for the ensuing drastic consequences.¹⁴

2.3 Scalar-graviton effective potential

The reduced Bianchi identity, $\nabla_\nu G_\mu^\nu = 0$, results in the continuity condition, $\nabla_\nu T_\mu^\nu = 0$, for the total energy-momentum tensor $T_{\mu\nu} = T_{s\mu\nu} + T_{m\mu\nu}$, to be presented as follows:

$$\partial_\mu W_s + W_s \partial_\mu \sigma + \nabla_\nu T_{m\mu}^\nu = 0. \quad (9)$$

With account for (8) the equation above is nothing but the third-order FE for σ , being quite complicated. However, it greatly simplifies if $T_{m\mu\nu}$ fulfills the continuity condition by itself, $\nabla_\nu T_{m\mu}^\nu = 0$. A priori, this would require some fine tuning. Still, if L_m is independent of σ (and thus of X), this condition is ensured without such a tuning as in GR due to GC (which now accounts only for the metric and matter, but not for X) supplemented by the

¹²Clearly, this FE encompasses as a marginal case the ordinary solutions for σ , with $\delta(L_m + L_s)/\delta \sigma = 0$, as well.

¹³Because the major part of the energy content of the Universe is attributed in the Λ CDM model to DM and DE, associating the latter ones with the vacuum X^a may realize in QMGR (with the potential term also dependent on X^a) a DC counterpart of the Mach’s principle (irrespective of the ordinary matter).

¹⁴Such an unconventional term appears in systogravity like a “Black Swan”. The latter may be said, after N. Taleb, to be a clue event/phenomenon producing drastic consequences hardly (if any) foreseeable beforehand, but, in principle, explainable afterwards.

matter FE. In this case (or under the absence of matter) one gets the solution to (9) as follows:

$$W_s = W_0 e^{-\sigma}, \quad (10)$$

with W_0 an integration constant. With account for (8) the scalar FE now becomes

$$\kappa_s^2 \nabla^\lambda \nabla_\lambda \sigma + \partial \bar{V}_s / \partial \sigma = 0, \quad (11)$$

where \bar{V}_s is the effective scalar potential:

$$\bar{V}_s \equiv V_s + W_0 e^{-\sigma}. \quad (12)$$

In distinction with the parameters of V_s , the value and sign of W_0 is not fixed ones forever, but is free to vary for the different solutions. Eventually, this arbitrariness allows one to consider the cosmic phenomena on the quite different distance scales starting from the scales of the scalar-modified black holes (BHs) up to the Universe as a whole (see, below).

At last, if L_m is independent of σ (and thus of X), one can, with account for (3) and (10), present FE (5) for X^a at $W_0 \neq 0$ in the form

$$\partial_\lambda (\sqrt{-g} e^{-\sigma} X_a^\lambda) = \partial_\lambda (\sqrt{-\gamma} X_a^\lambda) = 0, \quad (13)$$

which proves to be independent of $g_{\mu\nu}$. Thus, after finding from FEs the metric and σ , and extracting hereof $\sqrt{-\gamma}$, one can find the proper X^a up to a residual freedom consistent with the required γ . Such an ambiguity is insignificant in systogravity and can, in principle, be reduced afterwards in the full theory. The proposed minimal QMGR/systogravity is a theoretically consistent next-to-GR theory of gravity and, as such, may be tried phenomenologically to supersede GR, in particular, in cosmology.

3 Beyond Λ CDM per scalar graviton

3.1 Scalar graviton as dark component

Let the total scalar-graviton/systolon field $\sigma(t, \mathbf{x})$ in the cosmic coordinates $x^\mu = (t, \mathbf{x})$ (see, below) in the Universe be partiteted as

$$\sigma = \sigma_0(t) + \sum \Delta\sigma(t, \mathbf{x}). \quad (14)$$

Here σ_0 is an omnipresent in the whole Universe (background) field and $\Delta\sigma$ are the piece-wise (perturbation) fields corresponding to a cosmic DSs (such as the scalar-modified BHs, galaxies or the cluster of galaxies) appearing on the distance scales much less than that of the Universe. In the framework of systogravity, DE and DM are proposed to be treated as the supplementary manifestations of the same generic field σ as the universal DC in the different contexts, respectively, either as the spatially homogeneous $\sigma_0(t)$ or as the spatially inhomogeneous $\Delta\sigma(t, \mathbf{x})$.¹⁵ Being a part of the gravity field in line with the metric, $\Delta\sigma$ should be treated together with the latter in the process of the growth of perturbations in the Universe. To get an idea of a more detailed picture of σ as DC, we consider here only some simplified cases.

¹⁵A principle difference between the two parts of σ is that $\partial_\mu \sigma_0$ is taken to be time-like, whereas $\partial_\mu \Delta\sigma$ is supposed (at least presently) to be space-like (as for the static $\Delta\sigma(\mathbf{x})$).

3.2 Scalar graviton as dark matter

In the context of DM, we mention the quasi-stationary cosmic DSs in the present time on the time intervals much less than the Universe evolution time, neglecting by their temporal dependence. The simplest such DSs correspond to the static spherically symmetric fields $\Delta\sigma(r)$, where r is the radial distance from the spatial origin inside the respective DS, and $W_s = W_0 e^{-\Delta\sigma(r)}$, $W_0 \leq 0$. One may envisage the three particular cases of such the DSs (in neglect by the Lagrangian potential V_s)¹⁶ as follows [5].^{17,18}

(i) The simplest case is presented by the so-called *dark fractures* (DFs), which are the singular in the spatial origin (but regular at the spatial infinity) compact DSs corresponding to $W_0 = 0$.¹⁹ Being filled up exclusively by the scalar gravitons/systolons as DM, DFs are produced ultimately due to a singularity of the space itself in the spatial origin of the DFs. In a more general case, DFs are to be supplemented by the ordinary matter in their spatial origin.²⁰ Asymptotically, DFs mimic DHs of GR, with the total mass determined both by the matter and the scalar field. In the limit $\Delta\sigma = 0$, the DFs reduce to the ordinary BHs.

(ii) An opposite case is presented by the so-called *dark halos* (DHs), which are the regular in the spatial origin (but singular at the spatial infinity) extended DSs. They correspond to $W_0 < 0$, with the negative spontaneous term producing an attractive potential well. Designating $W_0 \equiv -\kappa_s^2/R_0^2$, one gets $\Delta\sigma \simeq (r/R_0)^2$ at $r < R_0$ and $\Delta\sigma \sim \ln(r/R_0)^2$ at $r > R_0$, with R_0 the soft-core scale.²¹ The equivalent DM energy density for such the static spherically symmetric DHs, $\rho_{DH} = \rho_s + 3p_s$, at the distances $r > R_0$ proves to be $\rho_{DH} \sim \kappa_s^2/r^2 > 0$, with the equivalent mass inside a large radius $R > R_0$ being $M \sim \kappa_s^2 R$. At that, the attractive gravity force acting at the distance R on a test body possessing a transverse rotation velocity v satisfies the relation $(\kappa_s/\kappa_g)^2/R \sim v^2/R$. The DHs are, in principle, apt to describe the anomalous rotation curves with the constant asymptotic velocity $v|_\infty \sim v_s \equiv \kappa_s/\kappa_g$.²²

(iii) An interpolating case is presented by the so-called *dark lacunas* (DLs), which are the compact-extended DSs corresponding to $W_0 < 0$, with DFs in the spatial origin supplemented by DHs at the spatial periphery. Being the typical DSs, the DLs reduce as the marginal cases either to DFs or to DHs depending on the relation between the central singularity and the soft core. By their spatial configuration, such the cosmic DSs may naturally serve as the “seeds” for the galaxies. Supplemented, in turn, by the ordinary matter, DLs may be expected to constitute the real galaxies on the various distance scales R_0 . At the

¹⁶The latter is assumed here to be negligible becoming important only to the cut-off of DSs on their periphery. An opposite case with the heavy scalar gravitons/systolons would require a special consideration.

¹⁷The rotating cosmic DSs, with the stationary axisymmetric metric $g_{\mu\nu}(r, \theta)$, θ the azimuthal angle, and the static scalar field $\Delta\sigma(r, \theta)$, could a priori be envisaged, too.

¹⁸To be more precise, the considered static solutions refer to the properly chosen spatial coordinate to be adjusted eventually to the cosmic ones.

¹⁹Under the latter condition, $\Delta\sigma$ is similar to an ordinary scalar field (but for the nature of the singularity in the spatial origin).

²⁰At that, a singularity in space may serve as a seed for the DF.

²¹This logarithmic growth should ultimately be cut-off by the scalar-graviton/systolon mass m_s , with $\lambda_s = 1/m_s \gg R_0$.

²²To get the realistic rotation profiles in the galaxies, the ordinary matter should also be accounted for. Nevertheless, under $v|_\infty \sim 10^{-3}$ and $\kappa_g \simeq 2.4 \times 10^{18}$ GeV, it is expected that $\kappa_s \sim 10^{15}$ GeV, which remarkably proves to be of the order of the GUT scale [5].

much larger R_0 , the DLs properly modified by matter should, in principle, have the similar bearing to the clusters of galaxies.²³ To achieve this, the freedom of arbitrary choosing the spontaneous terms $W_0 \sim -1/R_0^2$ is crucial. An ultimate ability (if any) to describe the realistic galaxies and the galaxy clusters by means of DLs with matter (and, conceivably, rotation) would be crucial for the minimal QMGR/systogravity.

3.3 Scalar graviton as dark energy

Universe evolution equations The FRW metric for the homogeneous isotropic Universe is given in the conventional notation by the line element

$$ds^2 = dt^2 - a^2 \left(\frac{1}{1 - Kr^2} dr^2 + r^2 d\Omega^2 \right), \quad (15)$$

where t is the cosmic standard time, r the radial distance from an (arbitrary chosen) spatial origin, $a(t)$ a scale factor, $K = k/l_0^2$, with l_0 an arbitrary fixed unit of length, and $k = 0, \pm 1$ for the spatially flat, closed and open Universe, respectively. Let the homogeneous and isotropic Universe be filled up by a continuous medium with the energy-momentum tensor

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}, \quad (16)$$

where ρ and p are, respectively, the energy density and pressure, and u^μ ($u^\lambda u_\lambda = 1$) the comoving four-velocity, with $u^\mu = (1, 0, 0, 0)$ in the cosmic standard coordinates. Then the Friedman-Lemaître equations for the evolution of the Universe look like

$$\begin{aligned} \ddot{a}/a &= -(\rho + 3p)/6\kappa_g^2, \\ H^2 + K/a^2 &= \rho/3\kappa_g^2, \end{aligned} \quad (17)$$

with $H \equiv \dot{a}/a$ being the Hubble parameter, and a dot meaning a time derivative.

To proceed further, we adopt first of all that the whole Universe is filled up by the homogeneous non-stationary scalar-graviton/systolon field $\sigma_0(t)$. At the energy scales greater than κ_s (but still lower than κ_g), the systogravity in the leading order in $\kappa_s^{-1}\partial/\partial t$ should be superseded by EFT accounting for all orders of the latter term (as well as dependent, possibly, on the temperature T_s), but still in the leading order in $\kappa_g^{-1}\partial/\partial t$. In the spirit of SCM, this stage should describe the primeval inflation. We have little to add to this point, but to accept it for granted. Nevertheless, after inflation, at the energies less than κ_s , the leading-order systogravity Lagrangian (2) should be (approximately) applicable. Associating the homogeneous scalar field $\sigma_0(t)$ with DE one can get (omitting here and in what follows the subscript for σ_0):²⁴

$$\rho_{DE}(p_{DE}) = \frac{1}{2}\kappa_s^2 \dot{\sigma}^2 \pm U_s, \quad (18)$$

In the above, one has $U_s = V_s + W_s$, with the scalar wave operator W_s as follows:

$$W_s = \kappa_s^2(\ddot{\sigma} + 3H\dot{\sigma}) + \partial V_s/\partial \sigma. \quad (19)$$

These expressions are valid at any k and correspond to the DE effective equation of state $\rho_{DE} = w_{DE}p_{DE}$, with the variable index $w_{DE}(\sigma)$. Under $\dot{\sigma} = 0$ (though, generally, $\ddot{\sigma} \neq 0$)

²³Some difference may be expected due to cut-offs.

²⁴The role of u^μ here plays $n^\mu = \nabla^\mu \sigma / (\nabla^\lambda \sigma \nabla_\lambda \sigma)^{1/2}$, which in the cosmic standard coordinates looks like $n^\mu = (1, 0, 0, 0)$. In fact, this may be taken in systogravity as a dynamical definition of u^μ .

one has $w_{DE} = -1$ mimicking thus a (σ -dependent) effective Λ -term. The appearance in ρ_{DE} and p_{DE} of the second time derivative of σ through W_s is quite unconventional feature peculiar to the scalar graviton/systolon due to dependence of σ on $g_{\mu\nu}$. Had σ been an ordinary scalar field, corresponding to $W_s = 0$, such an effect would be absent. For this reason, systogravity can not, generally, be reduced to GR plus an ordinary scalar field. Further, as in the Λ CDM model the ordinary matter m and DM, generically $M = (m, DM)$, concentrated in galaxies and the galaxy clusters, are to be smeared out homogeneously all over the Universe, too. At that, their energy densities and pressures are assumed satisfying the proper equations of state $p_m = w_m \rho_m$ and $p_{DM} = w_{DM} \rho_{DM}$, with the state indices w_m and w_{DM} , respectively. Due to the relative transparency of galaxies, p_{DM} may be assumed to be small, justifying thus the assumption of the cold or warm DM (at least at the relatively late stages of the evolution of the Universe).²⁵ Altogether, the total energy density and pressure, ρ and p , are given by the sum of the three generic fractions:

$$\begin{aligned}\rho &= \rho_m + \rho_{DM} + \rho_{DE} \equiv \rho_M + \rho_{DE}, \\ p &= p_m + p_{DM} + p_{DE} \equiv p_M + p_{DE},\end{aligned}\tag{20}$$

with $M = (m, DM)$ referring to the total matter.²⁶ This should be supplemented by the continuity condition for DE and the total matter M :

$$\dot{W}_s + W_s \dot{\sigma} = -(\dot{\rho}_M + 3H(\rho_M + p_M)),\tag{21}$$

which follows from the reduced Bianchi identity. Generally, this is the third-order equation for σ accounting, in particular, for the transition of DE into DM (and v.v.).

Eqs. (17)–(21) present the general systogravity scenario for the evolution of the homogeneous isotropic Universe filled up with the scalar gravitons/systolons and matter. Having found $\sigma(t)$ and $a(t)$ (and, thus, $\gamma(t)$) one can then get with account for (13) the affine quartet $X^A = (X^0, X^A)$, $A = 1, 2, 3$, looking at $k = 0$ like

$$X^0 = \int \sqrt{-\gamma} dt, \quad X^A = \delta_n^A x^n\tag{22}$$

($n = 1, 2, 3$), so that $X = \det(\partial_\lambda X^A) = \sqrt{-\gamma}$, as it should be. This determines, in particular, the cosmic affine time $\hat{t} \equiv X^0(t)$ satisfying $\sqrt{-\hat{\gamma}} d\hat{t} = \sqrt{-\gamma} dt$, with $\hat{\gamma} \equiv \gamma(\hat{t}) = -1$.

Effective cosmological constant A significant simplification occurs if the total matter $M = (m, DM)$ is covariantly conserved, with the r.h.s of (21) being zero.²⁷ Then it follows hereof that $W_s = W_0 e^{-\sigma}$, with W_0 an integration constant. With account for (19), this, in turn, implies that the scalar-graviton/systolon FE at any k are determined by the effective potential $\bar{V}_s = V_s + W_0 e^{-\sigma}$ as follows:

$$\kappa_s^2(\ddot{\sigma} + 3H\dot{\sigma}) + \partial \bar{V}_s / \partial \sigma = 0.\tag{23}$$

Let $\bar{\sigma}$ be the position of the minimum of the effective potential, $\partial \bar{V}_s / \partial \sigma|_{\bar{\sigma}} = 0$. Neglecting by $\dot{\sigma}$ and $\ddot{\sigma}$ reduces this FE to $\partial \bar{V}_s / \partial \sigma = 0$, meaning σ to be restricted by $\bar{\sigma}$. By this token,

²⁵Under the latter assumption, what concerns the scalar-graviton/systolon DE becomes, in essence, independent of the previously made statement that DM has the similar nature, and may be applicable in a more general context.

²⁶The more conventional types of DM (if any) are counted here formally as m .

²⁷This suffices for the following. Assuming the absence of interactions of the ordinary matter and DM, one may moreover consider their separate covariant conservation.

designating $\bar{V}_s|_{\bar{\sigma}} \equiv \kappa_g^2 \bar{\Lambda}_s$ and replacing in ρ_{DE} and p_{DE} the σ -dependent \bar{V}_s by the constant $\bar{V}_s|_{\bar{\sigma}}$, one arrives (assuming $w_{DM} = 0$) at the standard Λ CDM model, corresponding to the effective CC $\bar{\Lambda}_s$, so that

$$\bar{\rho}_{DE} = -\bar{p}_{DE} = \kappa_g^2 \bar{\Lambda}_s. \quad (24)$$

There are nevertheless two important caveats. First, being determined through an interplay of the given Lagrangian potential V_s and the spontaneous contribution W_0 , the effective CC is not a true fundamental parameter.²⁸ Second, the effective Λ CDM model is valid for σ only in a relatively narrow region around $\bar{\sigma}$, being superseded in a wider region by the systogravity extension to SCM. The deviations from SCM may be crucial when σ is far away from $\bar{\sigma}$. A number of uncertainties may however be envisaged, e.g., an omitted leakage of DE into DM (or v.v.), as well as the neglected temperature dependence of the theory, etc. Nevertheless, reproducing in a limit the standard Λ CDM model, systogravity hopefully provides a proper way to extend such the (conceivably, simplified) model.

Modified late expansion Restricting consideration by the late stage of the evolution of the Universe, we assume at this stage the dominance of DE by putting $\rho_M = p_M = 0$. With account for (17), Eq. (23) gets at $k = 0$ the autonomous form as follows:

$$\ddot{\sigma} + \sqrt{3} \left(\frac{1}{2} v_s^2 \dot{\sigma}^2 + \bar{V}_s / \kappa_g^2 \right)^{1/2} \dot{\sigma} + \kappa_s^{-2} \partial \bar{V}_s / \partial \sigma = 0, \quad (25)$$

where $v_s = \kappa_s / \kappa_g \ll 1$. This is the master equation for the evolution of the Universe due to the scalar-graviton/systolon DE. Having found hereof σ one then finds $H \equiv \dot{a}/a = \rho_s^{1/2} / \sqrt{3} \kappa_g \geq 0$, as well as the respective scale factor

$$a = a_0 \exp \frac{1}{\sqrt{3}} \int \left(\frac{1}{2} v_s^2 \dot{\sigma}^2 + \bar{V}_s / \kappa_g^2 \right)^{1/2} dt, \quad (26)$$

with a_0 an integration constant. To envisage the behaviour of H we note that from (17) at $k = 0$ one can find

$$\dot{H} = -\frac{1}{2} v_s^2 \dot{\sigma}^2 \leq 0, \quad (27)$$

independent of \bar{V}_s . This means, in particular, that the Hubble parameter H always monotonically decays approaching from above a constant value $\bar{H} \geq 0$, which however may depend on \bar{V}_s . At that, the Hubble horizon H^{-1} monotonically expands to \bar{H}^{-1} .

In distinction with DM, which is dominated by the kinetic scalar-graviton/systolon contribution and weakly depends on the potential, the respective DE may crucially depend on V_s . To be more specific, let us choose the quadratic V_s , so that

$$\bar{V}_s = \frac{1}{2} Q_s \sigma^2 + W_0 e^{-\sigma}, \quad (28)$$

where $Q_s \equiv m_s^2 \kappa_s^2$, with m_s the scalar-graviton/systolon mass. Considering the dependence of the minimum of \bar{V}_s on W_0 at a fixed Q_s one roughly gets for $\bar{\sigma}$ and $\bar{V}_s|_{\bar{\sigma}} \equiv \kappa_g^2 \bar{\Lambda}_s$:

$$\begin{aligned} \bar{\sigma} &\simeq W_0 / Q_s, \quad \kappa_g^2 \bar{\Lambda}_s \simeq W_0, & \text{at } W_0 / Q_s \ll 1, \\ \bar{\sigma} &\sim 1, \quad \kappa_g^2 \bar{\Lambda}_s \sim W_0 \sim Q_s, & \text{at } W_0 / Q_s \sim 1, \\ \bar{\sigma} &\simeq \ln W_0 / Q_s, \quad \kappa_g^2 \bar{\Lambda}_s \simeq \frac{1}{2} Q_s \ln^2 W_0 / Q_s, & \text{at } W_0 / Q_s \gg 1. \end{aligned} \quad (29)$$

²⁸In particular, one may envisage the situation when a counterpart of the Lagrangian CC given by $V_s|_{\min} \equiv \kappa_g^2 \bar{\Lambda}_s$, is zero, whereas the effective CC, $\bar{\Lambda}_s$, is finite due to $W_0 \neq 0$, solving thus partially the CC problem. This resembles the situation in Unimodular Relativity (UR), where CC is not the Lagrangian parameter Λ , but an integration constant Λ_0 appearing spontaneously at the level of FEs.

Clearly, $\sigma \equiv \bar{\sigma}$ is the exact solution to Eq. (25). Moreover, studying the latter in the phase plane $(\sigma, \dot{\sigma})$ shows that $(\bar{\sigma}, 0)$ is an attractor for the solutions which wind around the attractor approaching the latter asymptotically at $t \rightarrow \infty$. It then follows from (26) that at $k = 0$ there fulfills $\bar{a} = a_0 \exp \bar{H}t$, with $\bar{H} = (\bar{\Lambda}/3)^{1/2}$. This indicates the appearance in systogravity of the modified late inflation even under the absence of the true CC. Had any of W_0 or Q_s been zero the asymptotic inflation would not take place (under the assumed $V_s|_{\min} = \kappa_g^2 \Lambda_s = 0$). Having explained/assumed the exact vanishing of the true CC, one can eventually explain in the framework of systogravity the effective CC to be not exactly zero, solving thus partially the CC problem.²⁹

For completeness, the cosmic affine time for the attractor at $k = 0$ is $\hat{t} = \bar{X}^0 = \bar{t}_0 \exp 3\bar{H}t$, \bar{t}_0 an integration constant, or inversely $t = (3\bar{H})^{-1} \ln \hat{t}/\bar{t}_0$, so that $ds^2 = d\hat{t}^2 / (3\bar{H}\hat{t})^2 - (3e^{\bar{\sigma}} \bar{H}\hat{t})^{2/3} d\mathbf{x}^2$, where \bar{t}_0 is properly chosen to ensure $e^{\bar{\sigma}} = \sqrt{-\bar{g}}$ with $\sqrt{-\bar{g}} = 1$. In these terms, the exponential scale factor reduces to the power one, $\bar{a} \sim (3\bar{H}\hat{t})^{1/3}$. This implies, in particular, that the cosmic standard time t and the scale factor \bar{a} during the late stage of the Universe expansion decelerate in terms of \hat{t} . Thus using the (physically distinguished) comoving with the vacuum cosmic affine time as the reference one would drastically change the view on the evolution of the Universe.³⁰

4 Conclusion

It may be concluded that in framework of the minimal QMGR/systogravity the omnipresent scalar-graviton/systolon field filling up homogeneously the Universe, with the (quasi-)stationary lacunas of the field as the seeds for the galaxies or the cluster of galaxies, may, in principle, present a concise picture of the Universe. Reproducing in a limit the standard Λ CDM model, the ensuing in systogravity scenario may eventually supersede the standard one. In particular, there follows hereof an alternative approach to CC as the effective one determined by the interplay of the Lagrangian and spontaneous contributions to the effective scalar potential. More detailed investigations of systogravity are evidently needed to support/restrict the proposed extension to the SCM/ Λ CDM model and the alternative approach to CC. It seems that the proposed scenario for the evolution of the Universe ought to be eventually either confirmed or rejected. After all, studying the minimal QMGR/systogravity as the next-to-GR theory of gravity to reveal/exclude the scalar graviton/systolon as the new particle, which drastically influences the Universe, is worthy to pursue. Under success, this would pave the way towards the full QMGR and beyond.

Acknowledgement Thanks are due to I.Y. Plev for assistance with the computer calculations. The stimulating e-mail discussions with Gilles Cohen-Tannoudji on the Mach's principle and the CC problem are gratefully acknowledged.

²⁹It may be said that systogravity realizes for the Universe, dominated by the scalar-graviton/systolon DE, an interplay of the predetermination (through the Lagrangian V_s) and the occasion (though the spontaneous W_0). In particular, this would distinguish the late fates of the members of an ensemble of the "multiverses" (if any).

³⁰Adopting the affine time may be in concord with the Fock's insistence (in the context of a scalar field in GR) on the importance of choosing the proper coordinates in GR, as the GC geometrical theory, for better clarifying its physical content [6].

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